Total No. of Questions : 8] [Total No. of Printed Pages : 4

(1126)

B.A./B.Sc. (General) Vth Semester (0005) Examination

0443

MATHEMATICS

Paper : I

Analysis–I

Time : 3 Hours] **[Maximum Marks : 30**

Note :– Attempt *Five* questions in all, selecting at least *one* question from each Section. All questions carry equal marks.

Section–A

1. (a) Prove that the set $\{.....2^{-3}, 2^{-2}, 2^{-1}, 1, 2, 2^2, 2^3, ...\}$ is countable.

(b) If $0 < a < b$, show that :

$$\left|\int_a^b \frac{\sin x}{x}dx\right| \leq \frac{2}{a}$$

2. (a) If f is continuous on $[a, b]$, then $f \in R(x)$ on $[a, b]$.

(b) Give an example of a bounded function f defined on a closed interval such that $|f|$ is R-integrable but f is not.

3. (a) If f is R-integrable on $[a, b]$ and k is any real number, then k_f is also R-integrable and

$$\int_a^b (k_f)(x)dx = k\int_a^b f(x)\, dx$$

(b) Show that :

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}} dx = \frac{1}{(a+b)^m a^n} \beta(m,n)$$

4. (a) Prove that :

$$\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$$

(b) Show that :

$$\int_0^\infty \frac{x}{1+x^6} dx = \frac{\pi}{3\sqrt{3}}$$

Section–B

5. (a) Discuss the convergence of $\int_0^\infty \frac{x\tan^{-1}x}{(1+x^4)^{1/3}}dx$.

(b) Discuss the convergence of $\int_a^b \frac{dx}{(x-a)\sqrt{b-x}}$.

6. (a) If $\phi(x)$ is bounded and monotonic in $[a, \infty)$ and $\int_a^\infty f(x)dx$ is convergent at ∞, then prove that $\int_a^\infty f(x)\phi(x)dx$ is convergent at ∞.

(b) Show that $\int_0^\infty \frac{\sin^2 x}{x^2}dx$ is convergent. Also using $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$, show that $\int_0^\infty \frac{\sin^2 x}{x^2}dx = \frac{\pi}{2}$.

7. (a) Show that $\int_0^{\pi/2} \sin x . \log \sin x\, dx$ is convergent and its value is $\log \frac{2}{e}$.

(b) Show that $\int_0^{\infty} \frac{\sin ax \sin bx}{x} dx$ converges to $\frac{1}{2}$

$\log \left(\frac{a+b}{a-b}\right)$ where $a + b$ are positive reals.

8. By applying rule of differentiation under integral sign, prove the follwing :

(a) $\int_0^{\pi/2} \log\left(\frac{a+b\sin\theta}{a-b\sin\theta}\right) \operatorname{cosec} \theta \, d\theta = \pi \sin^{-1} \frac{b}{a}$.

(b) $\int_0^{\pi/2} \frac{\log(1+y\sin^2 x)}{\sin^2 x} dx = \pi\left[\sqrt{1+y} - 1\right]$.

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